Code No: 121AB **R15**

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B. Tech I Year Examinations, January/February - 2025
MATHEMATICS - I
(Common to CE, EEE, ME, ECE, CSE, EIE, IT, AME, MIE, PTM)

Time: 3 Hours **Max. Marks: 75**

Note: i) Question paper consists of Part A, Part B.
ii) Part A is compulsory, which carries 25 marks. In Part A, answer all questions.
iii) In Part B, Answer any one question from each unit. Each question carries 10 marks and may have a, b as sub questions.

PART - A

(25 Marks)

1.a) Find the rank of the matrix: $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$. [2]

b) Find the sum and product of Eigen values of $A = \begin{bmatrix} 6 & 8 \\ 8 & -6 \end{bmatrix}$. [3]

c) Discuss the application of Rolle's theorem to the function $f(x) = |x|$ in $[-1, 1]$. [2]

d) Find the Jacobian $\frac{\partial(u,v)}{\partial(x,y)}$, when $u = x\sin y,\ v = y\sin x$. [3]

e) Compute $\int_0^{\frac{\pi}{2}} \sin^6\theta \cos^7\theta\, d\theta$. [2]

f) Evaluate the double integral $\int_3^4 \int_1^2 \frac{dydx}{(x+y)^2}$. [3]

g) Define Exact differential equation with example. [2]

h) Find the particular integral of the differential equation $(D^2 + D)\, y = x^2 + 2x + 4$. [3]

i) State the second shifting theorem for Laplace transforms. [2]

j) Find $L^{-1}\left\{\frac{s+8}{s^2+4s+5}\right\}$. [3]

PART - B

(50 Marks)

2.a) Solve the following system of linear equations by using Gaussian elimination method $2x + y + z = 10;\ 3x + 2y + 3z = 18;\ x + 4y + 9z = 16$.

b) Find the Eigen values and Eigen vectors of the matrix $A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$. [5+5]

OR

3.a) Determine k such that the system of homogeneous equations

$2x+y+2z=0$;

$x+y+3z=0$;

$4x+3y+kz=0$;

has non-trivial solution. Find the non-trivial solution.

b) Verify Cayley-Hamilton theorem for the matrix $A=\begin{pmatrix} 1 & 4 & -3 \\ 0 & 3 & 1 \\ 0 & 2 & -1 \end{pmatrix}$ and hence find its inverse. [5+5]

4.a) Verify Rolle's theorem for $f(x)=(x-a)^m(x-b)^n$, where m, n are positive integers in $[a,b]$.

b) Using mean value theorem, prove that $(0<a<b<1)$, $\frac{b-a}{1+b^2}<\tan^{-1}b-\tan^{-1}a<\frac{b-a}{1+a^2}$.

Hence show that $\frac{\pi}{4}+\frac{3}{25}<\tan^{-1}\frac{4}{3}<\frac{\pi}{4}+\frac{1}{6}$. [5+5]

OR

5.a) If $u=x+y+z$, $uv=y+z$, $uvw=z$, show that $\frac{\partial(x,y,z)}{\partial(u,v,w)}=u^2v$.

b) Examine for extreme values: $f(x,y)=x^3+y^3+3xy$. [5+5]

6.a) Evaluate the integral $I=\iint_R y\,dx\,dy$, where R is the region bounded by the parabolas $y^2=4x$ and $x^2=4y$.

b) Find the volume of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$. [5+5]

OR

7.a) Evaluate $\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}}\frac{1}{\sqrt{1-x^2-y^2-z^2}}\,dz\,dy\,dx$.

b) Find the area bounded by the parabola $y=x^2$ and the line $y=2x+3$. [5+5]

8.a) Solve the differential equation $x\log x\frac{dy}{dx}+y=2\log x$.

b) Water at temperature 100 ^{0}C cools in 10 min to 80^{0}C in a room of temperature 25^{0}C. (i) Find the temperature of water after 20 min. (ii) when the temperature 40^{0}C? [5+5]

OR

9.a) Solve $\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} + 4\frac{dy}{dx} = e^{2x}$.

b) A circuit has in series an electromotive force given by $E = 200\cos 100\,t$ Volts, a resistor of 5 ohms , an inductor of 0.05 henrys, and a capacitor of 4×10^{-4} farads. If the initial current and the initial charge on the capacitor are both zero, find the charge on the capacitor at any time t > 0. [5+5]

10.a) Find the Laplace transform of $\cos^2(at)$.

b) Use Laplace transforms, Find the solution of the initial value problem $y'' + 4y' + 4y = 12t^2e^{-2t}$, $y(0) = 2$, $y'(0) = 1$. [3+7]

OR

11.a) Find the inverse Laplace transforms of the function $\frac{s}{(s^2+\omega^2)^2}$ using convolution.

b) Use Laplace transforms, Find the solution of the initial value problem $y'' + 6y' + 13y = e^{-t}$, $y(0) = 0$, $y'(0) = 4$. [4+6]

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